

# Four-nucleon contact interactions from holographic QCD

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**ABSTRACT:** We calculate the low energy constants of four-nucleon interactions in an effective chiral Lagrangian in holographic QCD. We start with a D4-D8 model to obtain meson-nucleon interactions and then integrate out massive mesons to obtain the four-nucleon interactions in 4D. We end up with two low energy constants at the leading order and seven of them at the next leading order, which is consistent with the effective chiral Lagrangian. The values of the low energy constants are evaluated with the first five Kaluza-Klein resonances.

**KEYWORDS:** Gauge/gravity duality, Chiral Lagrangian.

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## 1. Introduction

The four-nucleon interaction in the effective chiral Lagrangian was first introduced by Weinberg [1] to describe the short-range part of the nuclear force, see [2] and references therein for a recent review on nuclear forces from low-energy QCD via chiral effective field theory. Like pion and pion-nucleon interactions in chiral Lagrangian, the four-Fermi operators are accompanied by unknown coupling constants, called low energy constants (LECs). These

constants are calculable from Quantum Chromodynamics (QCD), in principle. However, in reality the LECs are determined by a fit to some experimental data or through some model-dependent calculations. Various LECs in the meson sector of a low-energy effective chiral lagrangian involving pseudoscalar fields only were determined with the resonance saturation hypothesis [3]; the assumption that dominant contributions to the LECs come from the dynamics of the low-lying resonances. In [4], the coupling constants of four-nucleon interactions are calculated with the resonance saturation assumption.

The four-nucleon terms are also important to understand the bulk nuclear matter property using the chiral Lagrangian. For instance, it was shown that the repulsive vector mean field in the Walecka model can be identified by the Four-Fermi interaction in chiral Lagrangian [5]. This indicates that the chiral Lagrangian with the four-nucleon terms could be satisfactorily describing the bulk property of nuclear matter as the Walecka model [6] does in the mean field approximation. The role of the four-Fermi interaction in establishing a bridge between the chiral quark-meson coupling model and chiral Lagrangian within the mean field approximation was discussed in [7]. The effect of the four-Fermi interaction on pion and kaon condensation was delved into in [8].

In this work we evaluate the LECs in a D4-D8 model with explicit bulk baryon fields [9, 10, 11]. We start from the 4D meson-baryon Lagrangian obtained in [9, 10, 11]. We integrate out massive mesons to obtain 4D relativistic four-nucleon contact interactions. Then, we take a non-relativistic limit to arrive at the LECs in the effective chiral Lagrangian. In the sense that integrated-out massive mesons determine the value of the LECs, our approach is similar to [4] based on the resonance saturation assumption. We consider up to  $Q^2$  order, where  $Q$  is a typical momentum of a system at hand. At the leading order ( $Q^0$ ) we have two LECs and seven of them at  $Q^2$  order. We evaluate the nine LECs with contributions from the first five Kaluza-Klein (KK) modes.

## 2. A D4/D8 holographic QCD and meson-nucleon couplings

Holography [12] is a conjectural property of string theory, whereby a strongly coupled large  $N_c$  gauge theory can be recast into a weakly coupled closed string theory. In practice, one starts with a D-brane configuration that contains large  $N_c$  gauge theory as its low energy sector, study how closed string theory view the configuration, and identify the degrees of freedom in the latter as the gauge-invariant objects (color-singlets) of the former. As such, we end up with an effective description involving objects like glueballs, mesons, and baryons in case of QCD.

A prototypical and probably the best candidate so far is D4/D8 model. In this configuration, one starts with  $N_c$  D4-branes, but now compactified on a thermal circle  $S^1$  of radius  $1/M_{KK}$ , where one requires anti-periodic boundary conditions on all fermions along the circle, breaking all supersymmetry. Four dimensional remnant is precisely flavorless  $U(N_c)$  QCD at lowest energy level. The holography views this as a ten-dimensional supergravity theory in the background of [13]

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2) + \left(\frac{R}{U}\right)^{3/2} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \quad (2.1)$$

with  $f(U) \equiv 1 - U_{KK}^3/U^3$ .  $x^\mu$  and  $\tau$  are the directions along which the D4-brane is extended. The most central quantity is  $M_{KK} = 2\pi/\delta\tau = 3U_{KK}^{1/2}/2R^{3/2}$ , which is not only the scale associated with the circle  $\tau$  but also both the curvature scale at  $U = U_{KK}$  and the scale associated with the internal  $S^4$  manifold.  $R$  is more directly related to the underlying string construction as  $R^3 = \pi g_s N_c l_s^3$  with the string coupling  $g_s$  and string length scale  $l_s$ .  $U$  is the holographic direction and bounded from below by the condition  $U \geq U_{KK}$ .

Taking large  $N_c$  limit naturally decouples  $U(1)$  part of  $U(N_c)$  in the usual  $1/N_c$  expansion sense, so we have  $SU(N_c)$  theory at hand. Analyzing the type IIA supergravity theory in this background, near  $U \sim U_{KK}$ , is supposed to be equivalent to studying  $SU(N_c)$  QCD without quarks at very low energy. However, one must understand how mesons and baryons are realized in this context, which will be the subjects of next two subsections.

## 2.1 Mesons

For mesons and baryons, one must add quarks to the above. It turns out that the right thing to do is to introduce D8 and  $\overline{D8}$  pairs of branes. With  $N_f$  pairs, QCD with  $N_f$  massless quarks emerges, although from the holographic side one only sees mesons. On the latter viewpoint, the D8's and the  $\overline{D8}$ 's are located at  $\tau = 0$  and  $\tau = \pi\delta\tau$ , so in fact identified at  $U = U_{KK}$ . This gluing represents the chiral symmetry breaking in a geometric manner, since what would have been  $U(N_f) \times U(N_f)$  symmetry becomes  $U(N_f)$  instead. This is the celebrated Sakai-Sugimoto model [14].

In the end, the meson spectrum and interactions are all realized as the lowest-lying flavor gauge theory on D8 branes. Instead of going through how this construction works, let us summarize the meson sector in terms of a five-dimensional  $U(N_f)$  flavor gauge theory on the worldvolume (modulo  $S^4$  which we are ignoring) of the now connected D8 and  $\overline{D8}$ ,

$$-\frac{1}{4} \int d^4x dw \frac{1}{e(w)^2} \text{tr} \mathcal{F}^2 + \frac{N_c}{24\pi^2} \int_{4+1} \omega_5(\mathcal{A}) \quad (2.2)$$

with the Chern-Simons 5-form,  $\omega_5(\mathcal{A})$ , and

$$\frac{1}{e(w)^2} = \frac{\lambda N_c}{108\pi^3} u(w) M_{KK} . \quad (2.3)$$

The holographic radial coordinate  $U$  is exchanged in favor of  $w$ , related to  $u = U/U_{KK}$  as,

$$\frac{2}{3} w M_{KK} = \pm \int_1^u dy / \sqrt{y^3 - 1} . \quad (2.4)$$

This five-dimensional action contains an infinite tower of spin 1 mesons and the pseudo-scalar field  $U$  as

$$\mathcal{A}_\mu(x; w) = i [U^{-1/2}, \partial_\mu U^{1/2}] / 2 + i \{U^{-1/2}, \partial_\mu U^{1/2}\} \psi_0(w) + \sum_n v_\mu^{(n)}(x) \psi_{(n)}(w) \quad (2.5)$$

with eigenfunctions  $\psi_{(n)}(w)$  along  $w$  directions. The gauge choice here is  $\mathcal{A}_w = 0$ , which although not entirely sensible suffices for the purpose here. The zero-mode part  $U(x)$  can

be thought of as the open Wilson line,  $U(x) = \exp(i \int_w \mathcal{A})$ , in generic gauge but here was gauge-transformed into  $\mathcal{A}_\mu$ .

The quadratic terms in (2.2) produces two type of terms in four dimensions

$$\begin{aligned} & \int d^4x \left( \frac{f_\pi^2}{4} \text{tr} (U^{-1} \partial_\mu U)^2 + \frac{1}{32e_{Skyme}^2} \text{tr} [U^{-1} \partial_\mu U, U^{-1} \partial_\nu U]^2 \right) \\ & + \int d^4x \sum_{n=1}^{\infty} \text{tr} \left\{ \frac{1}{2} (dv^{(n)})_{\mu\nu} (dv^{(n)})^{\mu\nu} + m_{(n)}^2 v_\mu^{(n)} v^{(n)\mu} \right\} + \dots, \end{aligned} \quad (2.6)$$

with  $f_\pi^2 = (g_{YM}^2 N_c) N_c M_{KK}^2 / 54\pi^4$  and  $1/e_{Skyme}^2 \simeq 61(g_{YM}^2 N_c) N_c / 54\pi^7$ . For real QCD,  $M_{KK}$  would be roughly  $M_{KK} \sim m_N \sim 0.94\text{GeV}$ , while  $f_\pi \sim 93\text{MeV}$ , and this requires  $(g_{YM}^2 N_c) N_c \sim 50$ . For  $N_c = 3$ , this gives  $\lambda = g_{YM}^2 N_c \simeq 17$ . Suppressed here are interaction terms among these mesons, cubic, quartic, and also Wess-Zumino-Witten term which comes from  $w_5(\mathcal{A})$ . This theory of holographic mesons has been investigated much in the literature. We emphasize that apart from the two input parameters  $\lambda$  and  $M_{KK}$ , no other tunable parameter exists. All the masses and all the couplings are fixed unambiguously via this Kaluza-Klein process from five dimensions to four dimensions. This remarkable aspect will persist to baryon sector in next subsection.

For later purpose, we wish to identify four-dimensional mesons more clearly. We assume  $SU(2)$  isospin symmetry and separate out the iso-singlet and the iso-triplet sector as

$$v_\mu^{(2k-1)} = \omega_\mu^{(k)} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} + \frac{1}{2} \rho_\mu^{(k)a} \tau^a, \quad v_\mu^{(2k)} = f_\mu^{(k)} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} + \frac{1}{2} a_\mu^{(k)a} \tau^a, \quad (2.7)$$

for vectors and axial-vectors, respectively. We will sometimes use the notation  $\rho_\mu = \rho_\mu^a \tau^a$  and  $a_\mu = a_\mu^a \tau^a$  also. Even/odd nature of  $\psi_{(n)}(w)$  translates to the usual parity of the corresponding mesons, so vectors,  $\rho$ 's and  $\omega$ 's, and axial vectors,  $a_1$ 's and  $f_1$ 's, alternates in the infinite tower of massive spin 1 mesons. On the other hand, Goldstone bosons associated with chiral symmetry breaking reside in the open Wilson line as

$$U = \exp(\pi i (\eta' + \pi^a \tau_a) / f_\pi). \quad (2.8)$$

The  $U(1)$  part  $\eta'$  picks up a mass term,

$$m_{\eta'}^2 = \frac{1}{27\pi^2} \frac{N_f}{N_c} \lambda^2 M_{KK}^2 \quad (2.9)$$

via a holographic version of axial anomaly. We refer readers to Ref. [14] for derivation of this additional contribution.

## 2.2 Holographic baryons

Baryons are identified as the soliton of the above flavor gauge theory, characterized by the first Pontryagin number,

$$p_1(\mathcal{F}) \equiv \frac{1}{8\pi^2} \int_{x^{1,2,3,w}} \text{tr} \mathcal{F} \wedge \mathcal{F} = 1. \quad (2.10)$$

For simplicity, let us consider  $N_f = 2$ . Quantizing the soliton to produce spin 1/2 baryons, and representing them via a local field  $\mathcal{B}$ , baryon dynamics can be added to the flavor gauge theory of meson sector as [9, 10]

$$+ \int d^4x dw \left[ -i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - im_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \frac{g(w)^2 \rho_{baryon}^2}{e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B} \right] \quad (2.11)$$

where

$$m_{\mathcal{B}}(w) = \frac{4\pi^2}{e(w)^2}, \quad \rho_{baryon}^2 \simeq \frac{(2 \cdot 3^7 \cdot \pi^2/5)^{1/2}}{M_{KK}^2 \lambda}, \quad (2.12)$$

and the function  $g_5(w)$  is known only at  $w = 0$ ,  $g_5(0) = 2\pi^2/3$ , which suffices for what follows in the large  $\lambda$  and the large  $N_c$  limits. We will not repeat how this action is derived from D4/D8 holographic QCD, but note again that there is no free parameter.

Note that the coupling between mesons and baryons are achieved via two interaction terms. The first is embedded into the covariant derivative,

$$D_m \equiv \partial_m - i(N_c \mathcal{A}_m^{U(1)} + A_m), \quad (2.13)$$

for which the flavor gauge field  $\mathcal{A}_m$  of (2.2) is decomposed as  $\mathcal{A}_m^{U(1)} + A_m$  with traceless  $2 \times 2$   $A_m$ . The second is through a direct coupling to the  $SU(N_f = 2)$  field strength,  $F$ , which can be traced to the fact that the soliton underlying the baryon carried the second Pontryagin number,  $p_1(F) = 1$ , whose configuration is self-dual magnetic fields in the four spatial directions.

### 2.3 Nucleon-meson dynamics and cubic couplings

To obtain meson-nucleon interactions we expand

$$\mathcal{B}(x^\mu, w) = \mathcal{N}_+(x^\mu) f_+(w) + \mathcal{N}_-(x^\mu) f_-(w), \quad (2.14)$$

where  $\gamma^5 \mathcal{N}_\pm = \pm \mathcal{N}_\pm$ . The profile functions  $f_\pm(w)$  satisfy

$$\begin{aligned} \partial_w f_+(w) + m_{\mathcal{B}}(w) f_+(w) &= m_{\mathcal{N}} f_-(w), \\ -\partial_w f_-(w) + m_{\mathcal{B}}(w) f_-(w) &= m_{\mathcal{N}} f_+(w) \end{aligned} \quad (2.15)$$

where  $m_{\mathcal{N}}$  is the nucleon mass in 4D. The 4D Dirac field for the nucleon is given by

$$\mathcal{N} = \mathcal{N}_+ + \mathcal{N}_-. \quad (2.16)$$

The eigenfunctions  $f_\pm(w)$  are normalized as

$$\int_{-w_{max}}^{w_{max}} dw |f_+(w)|^2 = \int_{-w_{max}}^{w_{max}} dw |f_-(w)|^2 = 1, \quad (2.17)$$

and the eigenvalue  $m_{\mathcal{N}}$  is the mass of the nucleon mode  $\mathcal{N}(x)$ . From (2.15), we get the second-order equations for  $f_\pm(w)$

$$[-\partial_w^2 - \partial_w m_{\mathcal{B}}(w) + m_{\mathcal{B}}(w)^2] f_+(w) = m_{\mathcal{N}}^2 f_+(w),$$

$$[-\partial_w^2 + \partial_w m_{\mathcal{B}}(w) + m_{\mathcal{B}}(w)^2] f_-(w) = m_{\mathcal{N}}^2 f_-(w). \quad (2.18)$$

There is a 1-1 mapping of eigenmodes with  $f_-(w) = \pm f_+(-w)$ , where the sign choice is related to the sign choice of  $m_{\mathcal{N}}$ . Due to the asymmetry under  $w \rightarrow -w$ ,  $f_+(w)$  tends to shift to the positive  $w$  side and the opposite happens for  $f_-(w)$ . In this work we will take the convention  $f_-(w) = f_+(-w)$ .

Inserting this into the action (2.11), we find the following structure of the four-dimensional nucleon action [11]

$$\int d^4x \mathcal{L}_4 = \int d^4x (-i\bar{\mathcal{N}}\gamma^\mu \partial_\mu \mathcal{N} - im_{\mathcal{N}}\bar{\mathcal{N}}\mathcal{N} + \mathcal{L}_{\text{vector}} + \mathcal{L}_{\text{axial}}), \quad (2.19)$$

where the couplings to mesons are

$$\begin{aligned} \mathcal{L}_{\text{vector}} = & - \sum_{k \geq 1} \frac{g_V^{(k)\text{triplet}}}{2} \bar{\mathcal{N}}\gamma^\mu \rho_\mu^{(k)} \mathcal{N} - \sum_{k \geq 1} \frac{N_c g_V^{(k)\text{singlet}}}{2} \bar{\mathcal{N}}\gamma^\mu w_\mu^{(k)} \mathcal{N} \\ & + \sum_{k \geq 1} \frac{g_{dV}^{(k)\text{triplet}}}{2} \bar{\mathcal{N}}\gamma^{\mu\nu} \partial_\mu \rho_\nu^{(k)} \mathcal{N} + \dots, \end{aligned} \quad (2.20)$$

and

$$\begin{aligned} \mathcal{L}_{\text{axial}} = & \frac{g_A^{\text{triplet}}}{2f_\pi} \bar{\mathcal{N}}\gamma^\mu \gamma^5 \partial_\mu \pi \mathcal{N} + \frac{N_c g_A^{\text{singlet}}}{2f_\pi} \bar{\mathcal{N}}\gamma^\mu \gamma^5 \partial_\mu \eta' \mathcal{N} \\ & - \sum_{k \geq 1} \frac{g_A^{(k)}}{2} \bar{\mathcal{N}}\gamma^\mu \gamma^5 a_\mu^{(k)\text{triplet}} \mathcal{N} - \sum_{k \geq 1} \frac{N_c g_A^{(k)\text{singlet}}}{2} \bar{\mathcal{N}}\gamma^\mu \gamma^5 f_\mu^{(k)} \mathcal{N} + \dots \end{aligned} \quad (2.21)$$

with  $\pi = \pi^a \tau^a$ . The ellipses denote quartic or higher terms. A notable feature here is that derivative couplings to spin 1 mesons exist only for  $\rho$  mesons. All others vanish as we will see presently. The pseudoscalar coupling can be alternatively written as

$$- \left( \frac{g_A^{\text{triplet}}}{2f_\pi} \times 2m_{\mathcal{N}} \right) \bar{\mathcal{N}}\gamma^5 \pi \mathcal{N} - \left( \frac{g_A^{\text{singlet}} N_c}{2f_\pi} \times 2m_{\mathcal{N}} \right) \bar{\mathcal{N}}\gamma^5 \eta' \mathcal{N} \quad (2.22)$$

using the on-shell condition of nucleons, which define  $g_{\pi\mathcal{N}\mathcal{N}}$  and  $g_{\eta'\mathcal{N}\mathcal{N}}$ . Note that we are considering two flavor case  $N_f = 2$ ;  $\eta'$  denotes the trace part of the pseudo-scalar, regardless of the number of flavors.

Let us take a closer look at 4D cubic couplings. Here we summarize the results from Refs. [9, 10, 11] following the notation in Ref. [11]. The 5D Lagrangian (2.11) generate 4D couplings via the minimal coupling

$$- \int dw \bar{\mathcal{B}}\gamma^m (N_c \mathcal{A}_m^{U(1)} + A_m) \mathcal{B}, \quad (2.23)$$

and the derivative coupling

$$\int dw g_5(w) \frac{\rho_{\text{baryon}}^2}{e^2(w)} \bar{\mathcal{B}}\gamma^{mn} F_{mn} \mathcal{B}, \quad (2.24)$$

with  $g_5(0) = 2\pi^2/3$  [10], upon mode-expanding the flavor gauge field and retaining the lowest-lying mode of  $\mathcal{B}$ . For instance, the latter generates two types of terms as

$$\begin{aligned}\gamma^{\mu\nu} F_{\mu\nu}(x, w) &= 2 \sum_n \psi_{(n)}(w) \gamma^{\mu\nu} \partial_\mu \left[ v_\nu^{(n)}(x) \Big|_{\text{iso-triplet}} \right], \\ \gamma^{5\mu} F_{5\mu}(x, w) &= -2 \sum_n (\partial_w \psi_{(n)}(w)) \gamma^\mu \gamma^5 \left[ v_\mu^{(n)} \Big|_{\text{iso-triplet}} \right],\end{aligned}\quad (2.25)$$

from which it is already clear that iso-singlet vectors  $w$  and  $f_1$  cannot have derivative couplings to the nucleon in this approximation. Integrating over  $w$  will then give cubic couplings as overlap integrals involving one  $\psi$  and two  $f_1$ 's.

It is convenient to define three set of numbers  $A_n$ ,  $B_n$  and  $C_n$  as

$$\begin{aligned}A_n &= \int_{-w_{max}}^{w_{max}} dw |f_+(w)|^2 \psi_{(n)}(w), \\ B_n &= \int_{-w_{max}}^{w_{max}} dw \left( g_5(w) \frac{\rho_{baryon}^2}{e^2(w)} \right) f_-^*(w) f_+(w) \psi_{(n)}(w), \\ C_n &= \int_{-w_{max}}^{w_{max}} dw \left( g_5(w) \frac{\rho_{baryon}^2}{e^2(w)} \right) |f_+(w)|^2 \partial_w \psi_{(n)}(w)\end{aligned}\quad (2.26)$$

from which all cubic couplings to mesons are constructed.  $B$ 's are in responsible for the derivative couplings. Choosing the phase of nucleon eigenfunction as  $f_+(w) = -f_-(-w)$ , and noting that  $\psi_{(n)}(w)$  is even/odd function when  $n$  is even/odd, respectively, we see that  $B_{2k} = 0$ . This leads to the result that  $a_1$  mesons have no derivative coupling to nucleons.

Sometimes additional  $\gamma^5$  is generated in terms originating from vector-like 5D coupling, because  $\psi_{(2k)}$  and  $\partial_w \psi_{(2k-1)}$  are odd functions of  $w$  and  $\gamma^5 \mathcal{N}_\pm = \pm \mathcal{N}_\pm$ . Taking all of these into account, one finds

$$\begin{aligned}g_V^{(k)triplet} &= A_{2k-1} + 2C_{2k-1}, \\ g_A^{(k)triplet} &= 2C_{2k} + A_{2k}, \\ g_{dV}^{(k)triplet} &= 2B_{2k-1},\end{aligned}\quad (2.27)$$

and

$$\begin{aligned}g_V^{(k)singlet} &= A_{2k-1}, \\ g_A^{(k)singlet} &= A_{2k}.\end{aligned}\quad (2.28)$$

Note that the vertices involving isospin singlet vector and axial-vector mesons are constructed only by minimal coupling terms. Cubic couplings to pseudo-scalars are determined similarly as

$$\begin{aligned}g_A^{triplet} &= 4C_0 + 2A_0, \\ g_A^{singlet} &= 2A_0.\end{aligned}\quad (2.29)$$



### 3. Four-nucleon contact interactions in chiral Lagrangian

In this section we summarize the four-nucleon contact interactions in an effective chiral Lagrangian which was first introduced by Weinberg [1] to describe the short-range part of the nuclear force. Like pion and pion-nucleon interactions in chiral Lagrangian, the four-Fermi operators are accompanied by LECs whose value are undetermined. These constants are calculable from QCD, in principle. However, in practice the LECs are determined by a fit to some experimental data or through some model-dependent calculations. In the next section, we will calculate the LECs using the meson-baryon vertex derived from the D4/D8/ $\overline{\text{D8}}$  model.

#### 3.1 Structures and LECs

In the conventional one-boson exchange (OBE) model of the nucleon-nucleon ( $NN$ ) force long range part is dominated by one-pion exchange, intermediate attraction is mostly given by a scalar meson, and short range interaction is described in terms of the vector meson exchange. Note that the scalar does not necessarily be the chiral partner of the pion, and its effect can be described by two-pion exchanges. In a modern approach for the  $NN$  force based on a chiral effective Lagrangian, (multi-) pion effects together with contact nucleon interactions replace the OBE picture. We focus on the four-nucleon contact interactions that can be expressed as a sum of local operators with increasing number of derivatives, or expansion in powers of a small momentum scale  $Q$ .

$O_S$	$(N^\dagger N)(N^\dagger N)$	leading	$O_S^\tau$	$(N^\dagger \tau^a N)(N^\dagger \tau^a N)$
$O_T$	$(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N)$	$(Q^0)$	$O_T^\tau$	$(N^\dagger \tau^a \boldsymbol{\sigma} N) \cdot (N^\dagger \tau^a \boldsymbol{\sigma} N)$
$O_1$	$(N^\dagger \nabla N)^2 + \text{h.c.}$	$\sigma \times 0$	$O_1^\tau$	$(N^\dagger \tau^a \nabla N)^2 + \text{h.c.}$
$O_2$	$(N^\dagger \nabla N) \cdot (\nabla N^\dagger N)$		$O_2^\tau$	$(N^\dagger \tau^a \nabla N) \cdot (\nabla N^\dagger \tau^a N)$
$O_3$	$(N^\dagger N)(N^\dagger \nabla^2 N) + \text{h.c.}$		$O_3^\tau$	$(N^\dagger \tau^a N)(N^\dagger \tau^a \nabla^2 N) + \text{h.c.}$
$O_4$	$i(N^\dagger \nabla N) \cdot (\nabla N^\dagger \times \boldsymbol{\sigma} N) + \text{h.c.}$	$\sigma \times 1$	$O_4^\tau$	$i(N^\dagger \tau^a \nabla N) \cdot (\nabla N^\dagger \tau^a \times \boldsymbol{\sigma} N) + \text{h.c.}$
$O_5$	$i(N^\dagger N)(\nabla N^\dagger \cdot \boldsymbol{\sigma} \times \nabla N)$		$O_5^\tau$	$i(N^\dagger \tau^a N)(\nabla N^\dagger \cdot \tau^a \boldsymbol{\sigma} \times \nabla N)$
$O_6$	$i(N^\dagger \boldsymbol{\sigma} N) \cdot (\nabla N^\dagger \times \nabla N)$		$O_6^\tau$	$i(N^\dagger \tau^a \boldsymbol{\sigma} N) \cdot (\nabla N^\dagger \tau^a \times \nabla N)$
$O_7$	$(N^\dagger \boldsymbol{\sigma} \cdot \nabla N)(N^\dagger \boldsymbol{\sigma} \cdot \nabla N) + \text{h.c.}$	$\sigma \times 2$	$O_7^\tau$	$(N^\dagger \tau^a \boldsymbol{\sigma} \cdot \nabla N)(N^\dagger \tau^a \boldsymbol{\sigma} \cdot \nabla N) + \text{h.c.}$
$O_8$	$(N^\dagger \sigma^i \partial_j N)(N^\dagger \sigma^j \partial_i N) + \text{h.c.}$		$O_8^\tau$	$(N^\dagger \tau^a \sigma^i \partial_j N)(N^\dagger \tau^a \sigma^j \partial_i N) + \text{h.c.}$
$O_9$	$(N^\dagger \sigma^i \partial_j N)(N^\dagger \sigma^i \partial_j N) + \text{h.c.}$		$O_9^\tau$	$(N^\dagger \tau^a \sigma^i \partial_j N)(N^\dagger \tau^a \sigma^i \partial_j N) + \text{h.c.}$
$O_{10}$	$(N^\dagger \boldsymbol{\sigma} \cdot \nabla N)(\nabla N^\dagger \cdot \boldsymbol{\sigma} N)$		$O_{10}^\tau$	$(N^\dagger \tau^a \boldsymbol{\sigma} \cdot \nabla N)(\nabla N^\dagger \tau^a \cdot \boldsymbol{\sigma} N)$
$O_{11}$	$(N^\dagger \sigma^i \partial_j N)(\partial_i N^\dagger \sigma^j N)$		$O_{11}^\tau$	$(N^\dagger \tau^a \sigma^i \partial_j N)(\partial_i N^\dagger \tau^a \sigma^j N)$
$O_{12}$	$(N^\dagger \sigma^i \partial_j N)(\partial_j N^\dagger \sigma^i N)$		$O_{12}^\tau$	$(N^\dagger \tau^a \sigma^i \partial_j N)(\partial_j N^\dagger \tau^a \sigma^i N)$
$O_{13}$	$(N^\dagger \sigma^i N)(\partial_j N^\dagger \sigma^j \partial_i N) + \text{h.c.}$		$O_{13}^\tau$	$(N^\dagger \tau^a \sigma^i N)(\partial_j N^\dagger \tau^a \sigma^j \partial_i N) + \text{h.c.}$
$O_{14}$	$2(N^\dagger \sigma^i N)(\partial_j N^\dagger \sigma^i \partial_j N)$		$O_{14}^\tau$	$2(N^\dagger \tau^a \sigma^i N)(\partial_j N^\dagger \tau^a \sigma^i \partial_j N)$

**Table 1:** Isosinglet operators

**Table 2:** Isotriplet operators

At the leading order (LO) ( $Q^0$ ), the expansion is the four (non-relativistic) nucleon

interactions with no derivatives [1]

$$\mathcal{L}^{(0)} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \boldsymbol{\sigma} N) \cdot (N^\dagger \boldsymbol{\sigma} N), \quad (3.1)$$

where  $N$  is the two component nucleon field and  $C_S$  and  $C_T$  denote the low energy constants (LECs). At  $Q^2$  order, the contact Lagrangian can be written as [15]

$$\mathcal{L}^{(2)} = -\sum_{i=1}^{14} C'_i O_i, \quad (3.2)$$

where  $C'_i$  are LECs and  $O_i$  are 14 operators listed in Table 1. Then, the four-point contact Lagrangian up to  $Q^2$  is

$$\mathcal{L} = -\frac{1}{2}C_S O_S - \frac{1}{2}C_T O_T - \sum_{i=1}^{14} C'_i O_i. \quad (3.3)$$

The 14 operators in Table 1 are the isosinglet ( $I = 0$ ) operators and the isotriplet ( $I = 1$ ) operators are in Table 2. Note that only 12 out of these 14 operators are independent since

$$O_7 - O_8 = 2O_{11} - 2O_{10}, \quad O_4 + O_5 = O_6. \quad (3.4)$$

Using the Fierz identity (Appendix B), we can rewrite each isotriplet operator in terms of isosinglet operators as

$$\begin{aligned} O_S^\tau &= O_S, \\ O_T^\tau &= 3O_S - 2O_T, \\ O_1^\tau &= O_1, \\ O_2^\tau &= -O_1 - 3O_2 - O_3, \\ O_3^\tau &= O_3, \\ O_4^\tau &= -3O_4, \\ O_5^\tau &= O_4 - O_5 - O_{10} + O_{11}, \\ O_6^\tau &= -O_4 - O_6 - O_{10} + O_{11}, \\ O_7^\tau &= O_1 + 2O_5 + 2O_6 + O_8 - O_9, \\ O_8^\tau &= O_1 - 2O_5 - 2O_6 + O_7 - O_9, \\ O_9^\tau &= 3O_1 - 2O_9, \\ O_{10}^\tau &= -\frac{1}{2}(O_1 + 2O_2 + O_3) - O_5 - O_6 - O_{10} + O_{13} - \frac{1}{2}O_{14}, \\ O_{11}^\tau &= -\frac{1}{2}(O_1 + 2O_2 + O_3) + O_5 + O_6 - O_{11} + O_{13} - \frac{1}{2}O_{14}, \\ O_{12}^\tau &= -\frac{3}{2}(O_1 + 2O_2 + O_3) - O_{12} - \frac{1}{2}O_{14}, \\ O_{13}^\tau &= 2O_2 + 2O_{10} + 2O_{11} - 2O_{12} - O_{13}, \\ O_{14}^\tau &= 6O_2 - 2O_{12} - O_{14}. \end{aligned} \quad (3.5)$$

We confirm the relation

$$O_7^\tau - O_8^\tau = 2O_{11}^\tau - 2O_{10}^\tau, \quad O_4^\tau + O_5^\tau = O_6^\tau \quad (3.6)$$

and again only 12 out of these 14 operators are independent for the isotriplet sector.

### 3.2 Non-relativistic limit

The relativistic fermion field  $\mathcal{N}$  can be reduced to the two-component spinor  $N$  via non-relativistic expansion. For this, it is convenient to use a Dirac basis

$$\gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

whereby we have an expansion

$$\mathcal{N}(x) = \begin{pmatrix} N(x) + \frac{1}{8m_{\mathcal{N}}^2} \nabla^2 N(x) \\ \frac{1}{2m_{\mathcal{N}}} \boldsymbol{\sigma} \cdot \nabla N(x) \end{pmatrix} + \mathcal{O}(Q^3). \quad (3.7)$$

In the leading  $Q^0$  order, this becomes

$$\mathcal{N}(x) = \begin{pmatrix} N(x) \\ 0 \end{pmatrix}. \quad (3.8)$$

Since we are to compute contact terms up to dimension eight operators, we retain higher order correction in the upper component. Details of this expansion is reviewed in Appendix A for the sake of completeness.

One consequence of building quartic contact terms from reduction of relativistic interaction vertices is that not all of  $O$ 's in Table 1 appears independently. The underlying Lorentz symmetry constrains the contact terms such that only nine (2+7) linearly independent combinations appear up to dimension eight. These are [16]

$$\begin{aligned} \mathcal{A}_S &= O_S + \frac{1}{4m_{\mathcal{N}}^2} (O_1 + O_3 + O_5 + O_6), \\ \mathcal{A}_T &= O_T - \frac{1}{4m_{\mathcal{N}}^2} (O_5 + O_6 - O_7 + O_8 + 2O_{12} + O_{14}), \\ \mathcal{A}_1 &= O_1 + 2O_2, \quad \mathcal{A}_2 = 2O_2 + O_3, \quad \mathcal{A}_3 = O_9 + 2O_{12}, \\ \mathcal{A}_4 &= O_9 + O_{14}, \quad \mathcal{A}_5 = O_5 - O_6, \\ \mathcal{A}_6 &= O_7 + 2O_{10}, \quad \mathcal{A}_7 = O_7 + O_8 + 2O_{13} \end{aligned} \quad (3.9)$$

which consist of two leading operators (of  $Q^0$  order) with higher order corrections and seven subleading ones (of  $Q^2$  order). The effective Lagrangian corresponding to (3.3) can be written as

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2} C_S \mathcal{A}_S - \frac{1}{2} C_T \mathcal{A}_T - \frac{1}{2} C_1 \mathcal{A}_1 + \frac{1}{8} C_2 \mathcal{A}_2 - \frac{1}{2} C_3 \mathcal{A}_3 \\ &\quad - \frac{1}{8} C_4 \mathcal{A}_4 - \frac{1}{4} C_5 \mathcal{A}_5 - \frac{1}{2} C_6 \mathcal{A}_6 - \frac{1}{16} C_7 \mathcal{A}_7. \end{aligned} \quad (3.10)$$

This is the same contact Lagrangian given in [15] in terms of  $O_{i=1\dots 14}$  operators in Table 1. The representation of seven independent coupling constants  $C_{1,\dots,7}$  of  $Q^2$  order Lagrangian is also in agreement with the result of [4, 17] by using the reparametrization invariance [18]. For isotriplet sector, we have

$$\mathcal{A}_S^T = O_S^T + \frac{1}{4m_{\mathcal{N}}^2} (O_1^T + O_3^T + O_5^T + O_6^T)$$

$$\begin{aligned}
&= O_S + \frac{1}{4m_{\mathcal{N}}^2}(O_1 + O_3 - O_5 - O_6 + O_7 - O_8) \\
&= \mathcal{A}_S + \frac{1}{4m_{\mathcal{N}}^2}(-2O_5 - 2O_6 + O_7 - O_8), \\
\mathcal{A}_T^\tau &= O_T^\tau - \frac{1}{4m_{\mathcal{N}}^2}(O_5^\tau + O_6^\tau - O_7^\tau + O_8^\tau + 2O_{12}^\tau + O_{14}^\tau) \\
&= 3O_S - 2O_T - \frac{1}{4m_{\mathcal{N}}^2}(-3O_1 - 3O_3 - 5O_5 - 5O_6 + 2O_7 - 2O_8 - 4O_{12} - 2O_{14}) \\
&= 3\mathcal{A}_S - 2\mathcal{A}_T, \\
\mathcal{A}_1^\tau &= O_1^\tau + 2O_2^\tau = -O_1 - 6O_2 - 2O_3 = -\mathcal{A}_1 - 2\mathcal{A}_2, \\
\mathcal{A}_2^\tau &= 2O_2^\tau + O_3^\tau = -2O_1 - 6O_2 - O_3 = -2\mathcal{A}_1 - \mathcal{A}_2, \\
\mathcal{A}_3^\tau &= O_9^\tau + 2O_{12}^\tau = -6O_2 - 3O_3 - 2O_9 - 2O_{12} - O_{14} = -3\mathcal{A}_2 - \mathcal{A}_3 - \mathcal{A}_4, \\
\mathcal{A}_4^\tau &= O_9^\tau + O_{14}^\tau = 3O_1 + 6O_2 - 2O_9 - 2O_{12} - O_{14} = 3\mathcal{A}_1 - \mathcal{A}_3 - \mathcal{A}_4, \\
\mathcal{A}_5^\tau &= O_5^\tau - O_6^\tau = -3O_5 + 3O_6 = -3\mathcal{A}_5, \\
\mathcal{A}_6^\tau &= O_7^\tau + 2O_{10}^\tau = -2O_2 - O_3 + O_8 - O_9 - 2O_{10} + 2O_{13} - O_{14} \\
&= -\mathcal{A}_2 - \mathcal{A}_4 - \mathcal{A}_6 + \mathcal{A}_7, \\
\mathcal{A}_7^\tau &= O_7^\tau + O_8^\tau + 2O_{13}^\tau = 2O_1 + 4O_2 + 3O_7 - O_8 - 2O_9 + 8O_{10} - 4O_{12} - 2O_{13} \\
&= 2\mathcal{A}_1 - 2\mathcal{A}_3 + 4\mathcal{A}_6 - \mathcal{A}_7. \tag{3.11}
\end{aligned}$$

In this work, we will construct the contact term by integrating out massive mesons in holographic QCD. We then obtain relativistic quartic operators after the integrating-out, such as  $\bar{\mathcal{N}}\mathcal{N}\bar{\mathcal{N}}\mathcal{N}$ ,  $\bar{\mathcal{N}}\gamma^\mu\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\mathcal{N}$ ,  $\bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}$ , and so on, and expand them into quartic operators involving  $N$ 's. Here we list the result of such expansion for all relativistic quartic operators we will encounter in the next section:

$$\begin{aligned}
\bar{\mathcal{N}}\gamma^\mu\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\mathcal{N} &\rightarrow -O_S + \frac{1}{4m_{\mathcal{N}}^2}(4O_2 + 2O_5 - 4O_6 - O_7 + O_9 - 2O_{10} + 2O_{12}) \\
&= -\mathcal{A}_S + \frac{1}{4m_{\mathcal{N}}^2}(\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + 3\mathcal{A}_5 - \mathcal{A}_6), \\
\bar{\mathcal{N}}\gamma^\mu\mathcal{N}\partial^2(\bar{\mathcal{N}}\gamma_\mu\mathcal{N}) &\rightarrow O_1 + 2O_2 = \mathcal{A}_1, \\
\bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N} &\rightarrow O_T + \frac{1}{4m_{\mathcal{N}}^2}(-2O_6 + O_7 - O_9 - 2O_{10} - 2O_{12} + 2O_{13} - 2O_{14}) \\
&= \mathcal{A}_T + \frac{1}{4m_{\mathcal{N}}^2}(-\mathcal{A}_4 + \mathcal{A}_5 - \mathcal{A}_6 + \mathcal{A}_7), \\
\bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N}\partial^2(\bar{\mathcal{N}}\gamma_\mu\gamma^5\mathcal{N}) &\rightarrow O_9 + 2O_{12} = \mathcal{A}_3, \\
\partial_\mu(\bar{\mathcal{N}}\gamma^\mu\gamma^5\mathcal{N})\partial_\nu(\bar{\mathcal{N}}\gamma^\nu\gamma^5\mathcal{N}) &\rightarrow O_7 + 2O_{10} = \mathcal{A}_6, \\
\bar{\mathcal{N}}\gamma_\mu\mathcal{N}\partial_\nu(\bar{\mathcal{N}}\gamma^{\nu\mu}\mathcal{N}) &\rightarrow \frac{1}{2m_{\mathcal{N}}}(O_1 + 2O_2 + 2O_5 - 2O_6 - O_7 + O_9 - 2O_{10} + 2O_{12}) \\
&= \frac{1}{2m_{\mathcal{N}}}(\mathcal{A}_1 + \mathcal{A}_3 - 2\mathcal{A}_5 - \mathcal{A}_6), \\
\partial_\nu(\bar{\mathcal{N}}\gamma^\nu_\mu\mathcal{N})\partial_\lambda(\bar{\mathcal{N}}\gamma^{\lambda\mu}\mathcal{N}) &\rightarrow -O_7 + O_9 - 2O_{10} + 2O_{12} = \mathcal{A}_3 - \mathcal{A}_6. \tag{3.12}
\end{aligned}$$

Here we used the equation of motion  $i\gamma^\mu\partial_\mu\mathcal{N} + im_{\mathcal{N}}\mathcal{N} = 0$  to eliminate time derivatives and performed partial integrations. Similarly we arrive at, for isotriplet sectors,

$$\bar{\mathcal{N}}\gamma^\mu\tau^a\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\tau^a\mathcal{N} \rightarrow -\mathcal{A}_S^\tau + \frac{1}{4m_{\mathcal{N}}^2}(\mathcal{A}_1^\tau + \mathcal{A}_2^\tau + \mathcal{A}_3^\tau + 3\mathcal{A}_5^\tau - \mathcal{A}_6^\tau)$$

$$\begin{aligned}
&= -\mathcal{A}_S + \frac{1}{4m_{\mathcal{N}}^2} (-3\mathcal{A}_1 - 5\mathcal{A}_2 - \mathcal{A}_3 - 9\mathcal{A}_5 + \mathcal{A}_6 - \mathcal{A}_7) , \\
&\bar{\mathcal{N}}\gamma^\mu\tau^a\mathcal{N}\partial^2 (\bar{\mathcal{N}}\gamma_\mu\tau^a\mathcal{N}) \rightarrow \mathcal{A}_1^\tau = -\mathcal{A}_1 - 2\mathcal{A}_2 , \\
&\bar{\mathcal{N}}\gamma^\mu\gamma^5\tau^a\mathcal{N}\bar{\mathcal{N}}\gamma_\mu\gamma^5\tau^a\mathcal{N} \rightarrow \mathcal{A}_T^\tau + \frac{1}{4m_{\mathcal{N}}^2} (-\mathcal{A}_4^\tau + \mathcal{A}_5^\tau - \mathcal{A}_6^\tau + \mathcal{A}_7^\tau) \\
&= 3\mathcal{A}_S - 2\mathcal{A}_T \\
&\quad + \frac{1}{4m_{\mathcal{N}}^2} (-\mathcal{A}_1 + \mathcal{A}_2 - \mathcal{A}_3 + 2\mathcal{A}_4 - 3\mathcal{A}_5 + 5\mathcal{A}_6 - 2\mathcal{A}_7) , \\
&\bar{\mathcal{N}}\gamma^\mu\gamma^5\tau^a\mathcal{N}\partial^2 (\bar{\mathcal{N}}\gamma_\mu\gamma^5\tau^a\mathcal{N}) \rightarrow \mathcal{A}_3^\tau = -3\mathcal{A}_2 - \mathcal{A}_3 - \mathcal{A}_4 , \\
&\partial_\mu (\bar{\mathcal{N}}\gamma^\mu\gamma^5\tau^a\mathcal{N}) \partial_\nu (\bar{\mathcal{N}}\gamma^\nu\gamma^5\tau^a\mathcal{N}) \rightarrow \mathcal{A}_6^\tau = -\mathcal{A}_2 - \mathcal{A}_4 - \mathcal{A}_6 + \mathcal{A}_7 , \\
&\bar{\mathcal{N}}\gamma_\mu\tau^a\mathcal{N}\partial_\nu (\bar{\mathcal{N}}\gamma^{\nu\mu}\tau^a\mathcal{N}) \rightarrow \frac{1}{2m_{\mathcal{N}}} (\mathcal{A}_1^\tau + \mathcal{A}_3^\tau - 2\mathcal{A}_5^\tau - \mathcal{A}_6^\tau) \\
&= \frac{1}{2m_{\mathcal{N}}} (-\mathcal{A}_1 - 4\mathcal{A}_2 - \mathcal{A}_3 + 6\mathcal{A}_5 + \mathcal{A}_6 - \mathcal{A}_7) , \\
&\partial_\nu (\bar{\mathcal{N}}\gamma^\nu{}_\mu\tau^a\mathcal{N}) \partial_\lambda (\bar{\mathcal{N}}\gamma^{\lambda\mu}\tau^a\mathcal{N}) \rightarrow \mathcal{A}_3^\tau - \mathcal{A}_6^\tau = -2\mathcal{A}_2 - \mathcal{A}_3 + \mathcal{A}_6 - \mathcal{A}_7 , \tag{3.13}
\end{aligned}$$

where we have ignored the higher order ( $m_{\mathcal{N}}^{-2}$ ) corrections of  $\mathcal{A}_S^\tau$  from (3.11).

#### 4. Four-nucleon contact interactions in holographic QCD

We now calculate the LECs in (3.10) in holographic QCD. We start from the 4D meson-baryon Lagrangian in (2.19) derived from the Sakai-Sugimoto model with explicit bulk baryon field [10, 11]. Then we integrate out massive mesons to obtain the values of the LECs. In the sense that integrated-out massive mesons determine the value of the LECs, our approach is similar to [4] based on the resonance saturation assumption.

##### 4.1 Isospin singlet mesons

We first consider the contributions from isospin singlet mesons to the LECs.

###### $\omega$ meson

For the isosinglet vector meson  $\omega$ , the interaction is shown in (2.20). The relativistic effective Lagrangian for the baryon field  $\mathcal{N}$  with  $\omega$  meson is

$$\begin{aligned}
& - \sum_{k \geq 1} \left( \frac{1}{4} (dw^{(k)})_{\mu\nu} (dw^{(k)})^{\mu\nu} + \frac{1}{2} m_{\omega^{(k)}}^2 \omega_\mu^{(k)} \omega^{(k)\mu} \right) \\
& - \sum_{k \geq 1} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right) \bar{\mathcal{N}} \gamma^\mu \omega_\mu^{(k)} \mathcal{N} , \tag{4.1}
\end{aligned}$$

where  $dw_{\mu\nu}^{(k)} = \partial_\mu \omega_\nu^{(k)} - \partial_\nu \omega_\mu^{(k)}$ . Solving the equation of motion for  $\omega^{(k)\mu}$ ,

$$(-m_{\omega^{(k)}}^2 + \partial^2) \omega_\mu^{(k)} - \partial_\mu \partial_\lambda \omega^{(k)\lambda} = \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right) \bar{\mathcal{N}} \gamma_\mu \mathcal{N} , \tag{4.2}$$

we find  $\partial_\mu \omega^{(k)\mu} = 0$  from the equations of motions of  $\mathcal{N}$  and  $\bar{\mathcal{N}}$ . Then by performing a derivative expansion, we obtain

$$\begin{aligned}\omega_\mu^{(k)} = & -\frac{1}{m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) singlet}}{2} \right) \bar{\mathcal{N}} \gamma_\mu \mathcal{N} \\ & - \frac{1}{m_{\omega^{(k)}}^4} \left( \frac{N_c g_V^{(k) singlet}}{2} \right) \partial^2 (\bar{\mathcal{N}} \gamma_\mu \mathcal{N}) + \mathcal{O}(Q^3).\end{aligned}\quad (4.3)$$

By substitution this into (4.1), we arrive at the contact interaction due to the  $\omega$  meson exchange

$$\begin{aligned}\mathcal{L}_\omega = & \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) singlet}}{2} \right)^2 \bar{\mathcal{N}} \gamma^\mu \mathcal{N} \bar{\mathcal{N}} \gamma_\mu \mathcal{N} \\ & + \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^4} \left( \frac{N_c g_V^{(k) singlet}}{2} \right)^2 \bar{\mathcal{N}} \gamma^\mu \mathcal{N} \partial^2 (\bar{\mathcal{N}} \gamma_\mu \mathcal{N}) + \mathcal{O}(Q^3).\end{aligned}\quad (4.4)$$

If we use the non-relativistic reduction (3.12), this becomes

$$\begin{aligned}\mathcal{L}_\omega \rightarrow & - \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) singlet}}{2} \right)^2 \mathcal{A}_S \\ & + \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) singlet}}{2} \right)^2 (\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + 3\mathcal{A}_5 - \mathcal{A}_6) \\ & + \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^4} \left( \frac{N_c g_V^{(k) singlet}}{2} \right)^2 \mathcal{A}_1.\end{aligned}\quad (4.5)$$

### $f_1$ meson

For the isospin singlet axial-vector meson  $f_1$ , the interaction is

$$- \sum_{k \geq 1} \left( \frac{N_c g_A^{(k) singlet}}{2} \right) \bar{\mathcal{N}} \gamma^\mu \gamma^5 f_\mu^{(k)} \mathcal{N}.\quad (4.6)$$

Similar to the  $\omega$  meson case, we integrate out  $f_1$  meson using the equation of motion and

$$\partial_\mu f^{(k)\mu} = -\frac{1}{m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) singlet}}{2} \right) \partial_\mu (\bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N})\quad (4.7)$$

to obtain

$$\begin{aligned}\mathcal{L}_{f_1} = & \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) singlet}}{2} \right)^2 \bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N} \bar{\mathcal{N}} \gamma_\mu \gamma^5 \mathcal{N} \\ & + \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^4} \left( \frac{N_c g_A^{(k) singlet}}{2} \right)^2 \bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N} \partial^2 (\bar{\mathcal{N}} \gamma_\mu \gamma^5 \mathcal{N})\end{aligned}$$

$$+ \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^4} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 \partial_\mu (\bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N}) \partial_\nu (\bar{\mathcal{N}} \gamma^\nu \gamma^5 \mathcal{N}) + \mathcal{O}(Q^3). \quad (4.8)$$

Again from (3.12), we get the non-relativistic reduced form

$$\begin{aligned} \mathcal{L}_{f_1} \rightarrow & \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 \mathcal{A}_T \\ & + \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 (-\mathcal{A}_4 + \mathcal{A}_5 - \mathcal{A}_6 + \mathcal{A}_7) \\ & + \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^4} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 (\mathcal{A}_3 + \mathcal{A}_6). \end{aligned} \quad (4.9)$$

$\eta'$  meson

For the pseudo-scalar meson  $\eta'$  case,

$$+ \left( \frac{N_c g_A^{(k) \text{singlet}}}{2f_\pi} \right) \bar{\mathcal{N}} \gamma^\mu \gamma^5 \partial_\mu \eta' \mathcal{N}, \quad (4.10)$$

the equations of motion of  $\eta'$  gives

$$\eta' = -\frac{1}{m_{\eta'}^2} \left( \frac{N_c g_A^{\text{singlet}}}{2f_\pi} \right) \partial_\mu (\bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N}) + \mathcal{O}(Q^3). \quad (4.11)$$

Then, the interaction can be reduced into the following four-point interactions

$$\mathcal{L}_{\eta'} = \frac{1}{2m_{\eta'}^2} \left( \frac{N_c g_A^{\text{singlet}}}{2f_\pi} \right)^2 \partial_\mu (\bar{\mathcal{N}} \gamma^\mu \gamma^5 \mathcal{N}) \partial_\nu (\bar{\mathcal{N}} \gamma^\nu \gamma^5 \mathcal{N}) + \mathcal{O}(Q^3). \quad (4.12)$$

This can be also written in a non-relativistic form as

$$\mathcal{L}_{\eta'} \rightarrow \frac{1}{2m_{\eta'}^2} \left( \frac{N_c g_A^{\text{singlet}}}{2f_\pi} \right)^2 \mathcal{A}_6. \quad (4.13)$$

#### Four-point contact Lagrangian for isospin singlet sector

We now summarize the non-relativistic four-point contact lagrangian from the isospin singlet mesons as

$$\mathcal{L}^{(I=0)} = \mathcal{L}_\omega + \mathcal{L}_{f_1} + \mathcal{L}_{\eta'}. \quad (4.14)$$

These are the four-point interaction with the low energy constants  $C_S$  and  $C_T$  of order  $Q^0$  and  $C_i$ 's of order  $Q^2$ . By direct comparison of (4.14) with the full effective Lagrangian (3.10), the leading order constants  $C_S$  and  $C_T$  have the structures

$$C_S^{(I=0)} = \sum_{k \geq 1} \frac{1}{m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2,$$

$$C_T^{(I=0)} = - \sum_{k \geq 1} \frac{1}{m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 \quad (4.15)$$

and the LECs of order  $Q^2$  are expressed as

$$\begin{aligned} -\frac{C_1^{(I=0)}}{2} &= \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2 + \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^4} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2, \\ \frac{C_2^{(I=0)}}{8} &= \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2, \\ -\frac{C_3^{(I=0)}}{2} &= \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2 + \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^4} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2, \\ -\frac{C_4^{(I=0)}}{8} &= -\frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2, \\ -\frac{C_5^{(I=0)}}{4} &= \frac{3}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2 + \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2, \\ -\frac{C_6^{(I=0)}}{2} &= -\frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k) \text{singlet}}}{2} \right)^2 - \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 \\ &\quad + \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^4} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2 + \frac{1}{2m_{\eta'}^2} \left( \frac{N_c g_A^{\text{singlet}}}{2f_\pi} \right)^2, \\ -\frac{C_7^{(I=0)}}{16} &= \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{f^{(k)}}^2} \left( \frac{N_c g_A^{(k) \text{singlet}}}{2} \right)^2. \end{aligned} \quad (4.16)$$

## 4.2 Isospin triplet mesons

We move on to the contribution from isospin triplet mesons to the LECs.

### $\rho$ meson

For the isospin triplet vector meson, the interaction is

$$- \sum_{k \geq 1} \left( \frac{g_V^{(k) \text{triplet}}}{2} \right) \bar{\mathcal{N}} \gamma^\mu \rho_\mu^{(k)a} \tau^a \mathcal{N} + \sum_{k \geq 1} \left( \frac{g_{dV}^{(k) \text{triplet}}}{2} \right) \bar{\mathcal{N}} \gamma^{\mu\nu} \partial_\mu \rho_\nu^{(k)a} \tau^a \mathcal{N}. \quad (4.17)$$

Again, solving the equation of motion for  $\rho^{(k)\mu a}$ , we arrive at

$$\begin{aligned} \rho_\mu^{(k)a} &= -\frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k) \text{triplet}}}{2} \right) \bar{\mathcal{N}} \gamma_\mu \tau^a \mathcal{N} - \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k) \text{triplet}}}{2} \right) \partial_\nu (\bar{\mathcal{N}} \gamma^\nu_\mu \tau^a \mathcal{N}) \\ &\quad - \frac{1}{m_{\rho^{(k)}}^4} \left( \frac{g_V^{(k) \text{triplet}}}{2} \right) \partial^2 (\bar{\mathcal{N}} \gamma_\mu \tau^a \mathcal{N}) + \mathcal{O}(Q^3). \end{aligned} \quad (4.18)$$



By substitution this into (4.17) we get the  $\rho$  meson exchange interaction

$$\begin{aligned}
\mathcal{L}_\rho = & \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 \bar{\mathcal{N}} \gamma_\mu \tau^a \mathcal{N} \bar{\mathcal{N}} \gamma^\mu \tau^a \mathcal{N} \\
& + \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^4} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 \bar{\mathcal{N}} \gamma_\mu \tau^a \mathcal{N} \partial^2 (\bar{\mathcal{N}} \gamma^\mu \tau^a \mathcal{N}) \\
& + \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) \bar{\mathcal{N}} \gamma_\mu \tau^a \mathcal{N} \partial_\nu (\bar{\mathcal{N}} \gamma^{\nu\mu} \tau^a \mathcal{N}) \\
& + \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k)triplet}}{2} \right)^2 \partial_\nu (\bar{\mathcal{N}} \gamma^\nu \tau^a \mathcal{N}) \partial_\lambda (\bar{\mathcal{N}} \gamma^{\lambda\mu} \tau^a \mathcal{N}) + \mathcal{O}(Q^3). \quad (4.19)
\end{aligned}$$

From (3.13), the non-relativistic reduction of this becomes

$$\begin{aligned}
\mathcal{L}_\rho \rightarrow & - \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 \mathcal{A}_S \\
& + \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 (-3\mathcal{A}_1 - 5\mathcal{A}_2 - \mathcal{A}_3 - 9\mathcal{A}_5 + \mathcal{A}_6 - \mathcal{A}_7) \\
& + \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^4} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 (-\mathcal{A}_1 - 2\mathcal{A}_2) \\
& + \frac{1}{2m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) (-\mathcal{A}_1 - 4\mathcal{A}_2 - \mathcal{A}_3 + 6\mathcal{A}_5 + \mathcal{A}_6 - \mathcal{A}_7) \\
& + \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k)triplet}}{2} \right)^2 (-2\mathcal{A}_2 - \mathcal{A}_3 + \mathcal{A}_6 - \mathcal{A}_7). \quad (4.20)
\end{aligned}$$

$a_1$  meson

For the isospin triplet axial-vector meson  $a_1$ , the interaction is

$$- \sum_{k \geq 1} \left( \frac{g_A^{(k)triplet}}{2} \right) \bar{\mathcal{N}} \gamma^\mu \gamma^5 a_\mu^{(k)a} \tau^a \mathcal{N}. \quad (4.21)$$

By integrating out  $a_1$  meson, we have,

$$\begin{aligned}
\mathcal{L}_{a_1} = & \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 \bar{\mathcal{N}} \gamma^\mu \gamma^5 \tau^a \mathcal{N} \bar{\mathcal{N}} \gamma_\mu \gamma^5 \tau^a \mathcal{N} \\
& + \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 \bar{\mathcal{N}} \gamma^\mu \gamma^5 \tau^a \mathcal{N} \partial^2 (\bar{\mathcal{N}} \gamma_\mu \gamma^5 \tau^a \mathcal{N}) \\
& + \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 \partial_\mu (\bar{\mathcal{N}} \gamma^\mu \gamma^5 \tau^a \mathcal{N}) \partial_\nu (\bar{\mathcal{N}} \gamma^\nu \gamma^5 \tau^a \mathcal{N}) + \mathcal{O}(Q^3). \quad (4.22)
\end{aligned}$$

From (3.13), the non-relativistic contact interaction term reads

$$\begin{aligned}
\mathcal{L}_{a_1} \rightarrow & \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 (3\mathcal{A}_S - 2\mathcal{A}_T) \\
& + \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 (-\mathcal{A}_1 + \mathcal{A}_2 - \mathcal{A}_3 + 2\mathcal{A}_4 - 3\mathcal{A}_5 + 5\mathcal{A}_6 - 2\mathcal{A}_7) \\
& + \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 (-4\mathcal{A}_2 - \mathcal{A}_3 - 2\mathcal{A}_4 - \mathcal{A}_6 + \mathcal{A}_7) .
\end{aligned} \tag{4.23}$$

#### Four-point contact Lagrangian for isotriplet sector

The non-relativistic four nucleon contact Lagrangian from the isospin triplet meson is then

$$\mathcal{L}^{(I=1)} = \mathcal{L}_\rho + \mathcal{L}_{a_1} . \tag{4.24}$$

Again, by direct comparison between (3.10) and (4.24) we obtain the leading order constants  $C_S$  and  $C_T$

$$\begin{aligned}
-\frac{1}{2}C_S^{(I=1)} &= -\sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 + 3 \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 , \\
-\frac{1}{2}C_T^{(I=1)} &= -2 \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 .
\end{aligned} \tag{4.25}$$

The LECs of order  $Q^2$  are given by

$$\begin{aligned}
-\frac{C_1^{(I=1)}}{2} &= -3 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 - \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^4} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 \\
&\quad - \frac{1}{2m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) - \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 , \\
\frac{C_2^{(I=1)}}{8} &= -5 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 - 2 \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^4} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 \\
&\quad - 4 \frac{1}{2m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) - 2 \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k)triplet}}{2} \right)^2 \\
&\quad + \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 - 4 \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 , \\
-\frac{C_3^{(I=1)}}{2} &= -\frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 - \frac{1}{2m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) \\
&\quad - \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k)triplet}}{2} \right)^2 - \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2
\end{aligned}$$

$$\begin{aligned}
& - \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2, \\
-\frac{C_4^{(I=1)}}{8} &= 2 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 - 2 \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2, \\
-\frac{C_5^{(I=1)}}{4} &= -9 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 + 6 \frac{1}{2m_{\mathcal{N}}} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) \\
& - 3 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2, \\
-\frac{C_6^{(I=1)}}{2} &= \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 + \frac{1}{2m_{\mathcal{N}}} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) \\
& + \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k)triplet}}{2} \right)^2 + 5 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 \\
& - \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2, \\
-\frac{C_7^{(I=1)}}{16} &= -\frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right)^2 - \frac{1}{2m_{\mathcal{N}}} \sum_{k \geq 1} \frac{1}{m_{\rho^{(k)}}^2} \left( \frac{g_V^{(k)triplet}}{2} \right) \left( \frac{g_{dV}^{(k)triplet}}{2} \right) \\
& - \sum_{k \geq 1} \frac{1}{2m_{\rho^{(k)}}^2} \left( \frac{g_{dV}^{(k)triplet}}{2} \right)^2 - 2 \frac{1}{4m_{\mathcal{N}}^2} \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 \\
& + \sum_{k \geq 1} \frac{1}{2m_{a^{(k)}}^4} \left( \frac{g_A^{(k)triplet}}{2} \right)^2. \tag{4.26}
\end{aligned}$$

## 5. Low energy constants at large $N_c$

Before we calculate the LECs numerically, we investigate the structure of them in the large  $\lambda$  and large  $N_c$  limit.

The leading large  $N_c$  and large  $\lambda$  scaling of coupling constants are: for pseudo-scalars ( $\varphi = \pi, \eta'$ )

$$\begin{aligned}
\frac{g_{\pi\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &= \frac{g_A^{triplet}}{2f_{\pi}} M_{KK} \simeq \frac{2 \cdot 3 \cdot \pi}{\sqrt{5}} \times \sqrt{\frac{N_c}{\lambda}}, \\
\frac{g_{\eta'\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &= \frac{N_c g_A^{singlet}}{2f_{\pi}} M_{KK} \simeq \sqrt{\frac{3^9}{2}} \pi^2 \times \frac{1}{\lambda N_c} \sqrt{\frac{N_c}{\lambda}}, \tag{5.1}
\end{aligned}$$

for vector mesons ( $v = \rho^{(k)}, \omega^{(k)}$ )

$$g_{\rho^{(k)}\mathcal{N}\mathcal{N}} = \frac{g_V^{(k)triplet}}{2} \simeq \sqrt{2 \cdot 3^3 \cdot \pi^3} \hat{\psi}_{(2k-1)}(0) \times \frac{1}{N_c} \sqrt{\frac{N_c}{\lambda}},$$

$$\begin{aligned}
g_{\omega^{(k)}\mathcal{N}\mathcal{N}} &= \frac{N_c g_V^{(k)\text{singlet}}}{2} \simeq \sqrt{2 \cdot 3^3 \cdot \pi^3} \hat{\psi}_{(2k-1)}(0) \times \sqrt{\frac{N_c}{\lambda}}, \\
\frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}}}{2m_{\mathcal{N}}} M_{KK} &= \frac{g_{dV}^{(k)\text{triplet}} M_{KK}}{2} \simeq \sqrt{\frac{2^2 \cdot 3^2 \cdot \pi^3}{5}} \hat{\psi}_{(2k-1)}(0) \times \sqrt{\frac{N_c}{\lambda}},
\end{aligned} \tag{5.2}$$

and for axial vector mesons  $(a_1^{(k)}, f_1^{(k)})$ ,

$$\begin{aligned}
g_{a^{(k)}\mathcal{N}\mathcal{N}} &\equiv \frac{g_A^{(k)\text{triplet}}}{2} \simeq \sqrt{\frac{2^2 \cdot 3^2 \cdot \pi^3}{5}} \hat{\psi}_{(2k)}'(0) \times \sqrt{\frac{N_c}{\lambda}}, \\
g_{f^{(k)}\mathcal{N}\mathcal{N}} &\equiv \frac{N_c g_A^{(k)\text{singlet}}}{2} \simeq \sqrt{\frac{3^9 \cdot \pi^5}{2}} \hat{\psi}_{(2k)}'(0) \times \frac{1}{\lambda N_c} \sqrt{\frac{N_c}{\lambda}}.
\end{aligned} \tag{5.3}$$

Therefore, the large  $N_c$  and large  $\lambda$  leading contributions arise from the following couplings

$$\frac{g_{\pi\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \sim g_{\omega^{(k)}\mathcal{N}\mathcal{N}} \sim \frac{\tilde{g}_{\rho^{(k)}\mathcal{N}\mathcal{N}} M_{KK}}{2m_{\mathcal{N}}} \sim g_{a^{(k)}\mathcal{N}\mathcal{N}} \sim \sqrt{\frac{N_c}{\lambda}}. \tag{5.4}$$

### 5.1 Large $N_c$ leading order with no derivative

In this limit, we consider the leading order of large  $N_c$  and the leading  $Q^2$  order for the four-point interactions. As in (5.4), the relevant mesons in this limit are  $\omega$ ,  $\rho$  and  $a_1$ .

#### Isospin singlet

In this  $Q^2 \rightarrow 0$  limit we ignore  $\partial^2$  terms and corresponding Dirac spinor  $\mathcal{N}$  is given in (3.8). For the isosinglet vector meson  $\omega$  case, therefore by (4.5), the  $\omega$  meson exchange interaction becomes

$$\mathcal{L}_{\omega} \rightarrow - \sum_{k \geq 1} \frac{1}{2m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k)\text{singlet}}}{2} \right)^2 O_S. \tag{5.5}$$

Then the four-point interaction from isosinglet sector takes the form

$$\mathcal{L}^{(I=0)} = \mathcal{L}_{\omega} = -\frac{1}{2} C_S O_S - \frac{1}{2} C_T O_T, \tag{5.6}$$

where

$$\begin{aligned}
C_S^{(I=0)} &= \sum_{k \geq 1} \frac{1}{m_{\omega^{(k)}}^2} \left( \frac{N_c g_V^{(k)\text{singlet}}}{2} \right)^2, \\
C_T^{(I=0)} &= 0.
\end{aligned} \tag{5.7}$$

#### Isospin triplet

We see from (5.4) that the dominant cubic couplings for the isotriplet vector meson  $\rho$  in the large  $N_c$  limit is the tensor coupling only. In this leading  $Q^2 \rightarrow 0$  limit, (4.20) shows that  $\rho$  meson part has no contribution. Therefore

$$\mathcal{L}_{\rho} \simeq 0 \tag{5.8}$$

in this limit. On the other hand, for the isotriplet axial vector meson  $a_1$ , (4.23) shows that the leading order  $a_1$  meson interaction is

$$\mathcal{L}_{a_1} \rightarrow \sum_{k \geq 1} \frac{1}{2m_{a(k)}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2 O_T. \quad (5.9)$$

Then we can write the leading  $Q^0$  order four-point interactions from the isotriplet mesons sector in the large  $N_c$  and large  $\lambda$  limit as

$$\mathcal{L}^{(I=1)} = \mathcal{L}_\rho + \mathcal{L}_{a_1} = -\frac{1}{2}C_S O_S - \frac{1}{2}C_T O_T \quad (5.10)$$

where

$$\begin{aligned} C_S^{(I=1)} &= 0, \\ C_T^{(I=1)} &= -\sum_{k \geq 1} \frac{1}{m_{a(k)}^2} \left( \frac{g_A^{(k)triplet}}{2} \right)^2. \end{aligned} \quad (5.11)$$

## 5.2 Large and finite $N_c$ up to $Q^2$ order

For the order of  $Q^2$  we include both  $\mathcal{L}^{(0)}$  and  $\mathcal{L}^{(2)}$  with the low energy constants (LECs)  $C_i$ 's. In the large  $N_c$  limit, the nucleon mass  $m_N$  is proportional to  $N_c$  and therefore terms with  $1/m_N^2$  or  $1/m_N$  in (4.16) and (4.26) will be suppressed relative to ones without such terms.

### Isospin singlet

For the large  $N_c$  limit, from (5.4) we need only

$$\mathcal{L}^{(I=0)} = \mathcal{L}_\omega \quad (5.12)$$

where  $\mathcal{L}_\omega$  does not include the leading  $1/4m_N^2$  term in (4.13). For a finite  $N_c$ , we have

$$\mathcal{L}^{(I=0)} = \mathcal{L}_\omega + \mathcal{L}_{f_1} + \mathcal{L}_{\eta'}, \quad (5.13)$$

which are given in (4.5), (4.9) and (4.13), respectively. The corresponding LECs are (4.15) and (4.16).

### Isospin triplet

For the large  $N_c$  limit, from (5.4) we need only

$$\mathcal{L}^{(I=1)} = \mathcal{L}_{\partial\rho} + \mathcal{L}_{a_1}, \quad (5.14)$$

where  $\mathcal{L}_{\partial\rho}$  refers to only the term with  $(g_{dV}^{(k)triplet})^2$  in (4.20), and  $\mathcal{L}_{a_N N}$  does not include the leading  $1/4m_N^2$  term from (4.23). For a finite  $N_c$ , we have

$$\mathcal{L}^{(I=1)} = \mathcal{L}_\rho + \mathcal{L}_{a_1}, \quad (5.15)$$

which are shown in (4.20) and (4.23), respectively and the corresponding LECs are given by (4.25) and (4.26).

## 6. Numerical results

Now we are at the stage of the numerical evaluation of the LECs for a finite  $N_c$  and  $\lambda$ . For this we need to calculate the masses and coupling constants in the 4D meson-baryon Lagrangian. For illustration purpose, we will take  $\lambda N_c = 50$  as in [9, 10, 11].

### 6.1 Gauge and baryonic profile functions

For efficient numerical estimates, we introduce dimensionless variables  $\tilde{w} = M_{KK}w$ ,  $\tilde{U} = U/U_{KK}$ , and  $\tilde{z} = z/U_{KK}$ . These are related as

$$\tilde{w} = \int_0^{\tilde{z}} \frac{d\tilde{z}}{[1 + \tilde{z}^2]^{2/3}} = \frac{3}{2} \int_1^{\tilde{U}} \frac{d\tilde{U}}{\sqrt{\tilde{U}^3 - 1}}. \quad (6.1)$$

In terms of these variables, we solve the eigenvalue equations [14]

$$-(1 + \tilde{z}^2)^{1/3} \partial_{\tilde{z}}((1 + \tilde{z}^2) \partial_{\tilde{z}} \psi_{(n)}) = \lambda_n \psi_{(n)} \quad (6.2)$$

for the gauge sector profile functions  $\psi_{(n)}$ . To compute (2.26), we introduce re-scaled gauge profile function  $\hat{\psi}_{(n)}(\tilde{w})$  that is defined as [11]

$$\hat{\psi}_{(n)}(\tilde{w}) = \sqrt{\frac{216\pi^3}{\lambda N_c}} \psi_{(n)}(w). \quad (6.3)$$

In addition, the conventional choice of the zero mode gauge field profile function can be  $\partial_{\tilde{z}} \psi_{(0)}(0) = 1/\pi$  and we define

$$\hat{\psi}_{(0)}(\tilde{z}) = \frac{1}{\pi} \tan^{-1} \tilde{z}. \quad (6.4)$$

For the baryonic sector, we have

$$m_{\mathcal{B}}(z) = m_{\mathcal{N}}^{(0)} \cdot \tilde{U} + m_0^e = M_{KK} \cdot \left( \frac{\lambda N_c}{27\pi} \tilde{U}(\tilde{w}) + \epsilon N_c \right) \quad (6.5)$$

with  $\epsilon \equiv \sqrt{2/15} \simeq 0.37$ . After dividing (2.18) by  $M_{KK}^2$ , we reach the eigenvalue equation for baryonic profile functions

$$\left[ -\partial_{\tilde{w}}^2 - \frac{\lambda N_c}{27\pi} \partial_{\tilde{w}} \tilde{U}(\tilde{w}) + \left( \frac{\lambda N_c}{27\pi} \tilde{U}(\tilde{w}) + \epsilon N_c \right)^2 \right] f_{+}(\tilde{w}) = \left( \frac{m_{\mathcal{N}}}{M_{KK}} \right)^2 f_{+}(\tilde{w}). \quad (6.6)$$

Then by solving (6.2) and (6.6), we can compute (2.26) and finally get the value of LECs.

### 6.2 Tower of couplings

The baryon wave function (2.11) is effectively localized at  $w \simeq 0$  in the large  $N_c$  and large  $\lambda$  limit. The coefficient function for the derivative tensor coupling  $\bar{\mathcal{B}}\gamma \cdot F\mathcal{B}$  has the central value near  $w \simeq 0$  as

$$g_5(w) \frac{\rho_{baryon}^2}{e^2(w)} \simeq 0.18 \frac{N_c}{M_{KK}}. \quad (6.7)$$

Then the three numbers  $A_n$ ,  $B_n$  and  $C_n$  in (2.26) become

$$\begin{aligned}
A_n &= \int_{-\infty}^{\infty} \frac{d\tilde{z}}{(1+\tilde{z}^2)^{2/3}} |f_+(\tilde{z})|^2 \psi_{(n)}(\tilde{z}), \\
B_n &= \frac{1}{M_{KK}} \cdot 0.18 N_c \int_{-\infty}^{\infty} \frac{d\tilde{z}}{(1+\tilde{z}^2)^{2/3}} f_-^*(\tilde{z}) f_+(\tilde{z}) \psi_{(n)}(\tilde{z}), \\
C_n &= 0.18 N_c \int_{-\infty}^{\infty} d\tilde{z} |f_+(\tilde{z})|^2 \partial_{\tilde{z}} \psi_{(n)}(\tilde{z})
\end{aligned} \tag{6.8}$$

where  $d\tilde{w} = d\tilde{z}/(1+\tilde{z}^2)^{2/3}$ . From (6.4) we also have  $g_A^{singlet} = 2A_0 = 0.137$ .

In section 2.1, we have mentioned that  $(g_{YM}^2 N_c) N_c \sim 50$  is required for  $M_{KK} \sim m_{\mathcal{N}} \sim 0.94 \text{ GeV}$ , and for the case of  $N_c = 3$ , this gives  $\lambda = g_{YM}^2 N_c \simeq 17$ . Now if we take  $\lambda N_c = 50$  with  $N_c = 3$ , we obtain the numerical eigenvalue  $m_{\mathcal{N}}/M_{KK} = 1.91$  from (6.6). (For other choices of  $\lambda N_c$  and the corresponding eigenvalues, see [10].) But to obtain LECs, we will take more realistic  $m_{\mathcal{N}} \simeq M_{KK}$ , since the ratio  $m_{\mathcal{N}}/M_{KK}$  is meaningful just as an eigenvalue of each  $\lambda N_c$  input. The resulting  $A_n$ ,  $B_n$ ,  $C_n$  and the coupling constants are shown in Table 3.

The KK towers of meson masses are listed in Table 4 from the previous work [11] and the mass of  $\eta'$  is given by (2.9). In addition,

$$f_\pi = \sqrt{\frac{\lambda N_c}{54\pi^4}} M_{KK} = 0.0975 M_{KK} \tag{6.9}$$

from [14].

$k$	$A_{2k-1}$ $=g_V^{(k)singlet}$	$B_{2k-1}$	$C_{2k-1}$	$A_{2k}$ $=g_A^{(k)singlet}$	$C_{2k}$	$A_{2k-1}+2C_{2k-1}$ $=g_V^{(k)triplet}$	$2B_{2k-1}$ $=g_{dV}^{(k)triplet}$	$2C_{2k}+A_{2k}$ $=g_A^{(k)triplet}$
1	5.93	3.05	-0.409	1.12	2.35	5.11	6.10	5.81
2	-3.21	-1.75	1.01	-1.03	-1.98	-1.19	-3.49	-4.98
3	1.25	0.756	-0.967	0.521	0.863	-0.685	1.51	2.25
4	-0.305	-0.222	0.481	-0.149	-0.188	0.658	-0.443	-0.526
5	0.0401	0.0392	-0.116	0.0206	0.0160	-0.191	0.0783	0.0526

**Table 3:** The values of the couplings.  $A_n$ ,  $C_n$  are dimensionless and  $B_n$  are of dimension  $M_{KK}^{-1}$ .

$k$	$m_{\omega^{(k)}} = m_{\rho^{(k)}}$	$m_{a^{(k)}} = m_{f^{(k)}}$
1	0.818	1.25
2	1.69	2.13
3	2.57	3.00
4	3.44	3.87
5	4.30	4.73

**Table 4:** The tower of meson masses in unit of  $M_{KK}$ .

### 6.3 Low energy constants

Now we obtain the values of the LECs at finite  $N_c$  from the first five towers of couplings (Table 3) and the corresponding meson masses (Table 4). The leading order ( $Q^0$ ) LECs,  $C_S$  and  $C_T$  of isosinglet and isotriplet channels are given in (4.15) and (4.25), respectively. In the next leading order ( $Q^2$ ), the LECs of isosinglet and isotriplet channels are listed in (4.16) and (4.26). The resulting values of the LECs are listed in Table 5, where we sum over the first five towers ( $k = 1, \dots, 5$ ).<sup>1</sup>

	$(I = 0)$	<i>From</i> $(I = 1)$	<i>Total</i>
$C_S$ ( $10^{-4} \text{ MeV}^{-2}$ )	1.44	-0.123	1.32
$C_T$ ( $10^{-4} \text{ MeV}^{-2}$ )	-0.0272	0.156	0.129
$C_1$ ( $10^{-9} \text{ MeV}^{-4}$ )	-0.270	0.0454	-0.225
$C_2$ ( $10^{-9} \text{ MeV}^{-4}$ )	0.159	-0.678	-0.379
$C_3$ ( $10^{-9} \text{ MeV}^{-4}$ )	-0.0414	0.0446	0.0032
$C_4$ ( $10^{-9} \text{ MeV}^{-4}$ )	0.00302	0.0213	0.0243
$C_5$ ( $10^{-9} \text{ MeV}^{-4}$ )	-0.240	-0.113	-0.353
$C_6$ ( $10^{-9} \text{ MeV}^{-4}$ )	0.0311	-0.0436	-0.0125
$C_7$ ( $10^{-9} \text{ MeV}^{-4}$ )	-0.00603	0.297	0.291

**Table 5:** The low energy constants from isospin singlet ( $I = 0$ ) and Fierz-transformed isospin triplet ( $I = 1$ ) sectors.

## 7. Summary

We evaluated the low energy constant of four-nucleon contact interactions in an effective chiral Lagrangian in the framework of holographic QCD. These contact interactions are essential to describe the short-range nuclear force and also crucial to understand the bulk nuclear matter properties in the chiral Lagrangian. We considered the Sakai-Sugimoto model with the bulk baryon field to obtain meson-nucleon interactions, and then we integrated out massive mesons to obtain the four-nucleon interactions in 4D. We obtained two independent contact terms at the leading order ( $Q^0$ ) and seven of them at the next leading order ( $Q^2$ ), which is consistent with the effective chiral Lagrangian. We calculated the values of the LECs with the first five Kaluza-Klein resonances.

It will be interesting to study some phenomenological consequences of the LECs determined in this work. An immediate example might be to study observables in the nucleon-nucleon scattering such as phase shifts.

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<sup>1</sup>In Ref. [9], a subleading correction that shifts  $N_c \rightarrow N_c + 2$  for certain types of cubic meson-nucleon couplings was considered and found to produce phenomenologically viable axial couplings to pion as well as the anomalous magnetic moment of nucleons. In the past, this shift has been shown to exist for operators associated with the Hedgehog configuration of Skyrmions [19], which in our approach manifest itself in  $\vec{B}\gamma \cdot F\vec{B}$  coupling in (2.11). In Appendix C, we list modified results for the LECs, assuming such a shift.



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## A. Non-relativistic Dirac spinor with relativistic correction

The free Dirac Lagrangian in our case is

$$\mathcal{L} = \bar{\mathcal{N}}(-i\gamma^\mu\partial_\mu - im_{\mathcal{N}})\mathcal{N} \quad (\text{A.1})$$

with the Weyl basis defined in [10]. To develop the expressions of some Dirac spinors with upper and lower parts, we can apply a unitary transformation  $\mathcal{N} \rightarrow \mathcal{U}\mathcal{N}$  and  $\gamma^\mu \rightarrow \mathcal{U}\gamma^\mu\mathcal{U}^\dagger$  then the Lagrangian is invariant under the transformation. If we adopt a unitary transformation

$$\mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ i & -1 \end{pmatrix},$$

then the transformed Dirac basis corresponding to (A.1) is

$$\gamma^0 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (\text{A.2})$$

Here we manipulate the decoupled two-component Dirac spinor field that is, for positive energies, the upper components are large and the lower are small. We use the Dirac-Pauli representation of the gamma matrices derived above and follow the non-relativistic reduction manipulation in [20]. Dirac's equation for a free electron is

$$i\frac{\partial}{\partial t}\mathcal{N} = (\boldsymbol{\alpha} \cdot \mathbf{p} + im_{\mathcal{N}}\beta)\mathcal{N} \quad (\text{A.3})$$

where

$$\alpha^i = \begin{pmatrix} 0 & -i\sigma^i \\ i\sigma^i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}. \quad (\text{A.4})$$

The the particle has its rest energy  $m_{\mathcal{N}}$  and this should be excluded in the non-relativistic approximation, and we denote  $\mathcal{N}$  by a function  $\mathcal{N}'$  with

$$\mathcal{N} = e^{-im_{\mathcal{N}}t}\mathcal{N}'. \quad (\text{A.5})$$

Then we have

$$\left(i\frac{\partial}{\partial t} + m_{\mathcal{N}}\right)\mathcal{N}' = (\boldsymbol{\alpha} \cdot \mathbf{p} + im_{\mathcal{N}}\beta)\mathcal{N}'. \quad (\text{A.6})$$

We can write

$$\mathcal{N}' = \begin{pmatrix} N \\ h \end{pmatrix}. \quad (\text{A.7})$$

In the non-relativistic limit ( $v \rightarrow 0$ ) the two components  $h$  vanish and this leads to an approximate equation involving  $N$  only. Then by substituting, we obtain the equations

$$i \frac{\partial}{\partial t} N = -i \boldsymbol{\sigma} \cdot \mathbf{p} h, \quad (\text{A.8})$$

$$i \frac{\partial}{\partial t} h + 2m_{\mathcal{N}} h = i \boldsymbol{\sigma} \cdot \mathbf{p} N. \quad (\text{A.9})$$

In the first approximation of  $h$ , the term  $2m_{\mathcal{N}} h$  is dominant on the left hand side of (A.9), and this gives

$$h = i \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{2m_{\mathcal{N}}} N = \frac{\boldsymbol{\sigma} \cdot \nabla}{2m_{\mathcal{N}}} h, \quad (\text{A.10})$$

or

$$\begin{pmatrix} 0 \\ h \end{pmatrix} = -\frac{1}{2m_{\mathcal{N}}} \gamma^i \partial_i \begin{pmatrix} N \\ 0 \end{pmatrix}. \quad (\text{A.11})$$

Substitution of this into (A.8) then gives the Pauli's equation. Let us now drive the second approximation for  $N$ . The density is

$$\rho = N^\dagger N + h^\dagger h = N^\dagger N + \frac{1}{4m_{\mathcal{N}}^2} (\nabla N^\dagger \cdot \boldsymbol{\sigma}) (\boldsymbol{\sigma} \cdot \nabla N). \quad (\text{A.12})$$

To get the wave equation corresponding to the Schödinger equation, one must assign  $N$  by another two-component field  $N_{\text{NR}}$  such that

$$\int d^3x N_{\text{NR}}^\dagger N_{\text{NR}} = \int d^3x \left\{ N^\dagger N + \frac{1}{4m_{\mathcal{N}}^2} (\nabla N^\dagger \cdot \boldsymbol{\sigma}) (\boldsymbol{\sigma} \cdot \nabla N) \right\} \quad (\text{A.13})$$

is the time independent integral. By the integration by parts, this can be rewritten as

$$\int d^3x N_{\text{NR}}^\dagger N_{\text{NR}} = \int d^3x \left\{ N^\dagger N - \frac{1}{8m_{\mathcal{N}}^2} \left( N^\dagger \nabla^2 N + (\nabla^2 N^\dagger) N \right) \right\}. \quad (\text{A.14})$$

Then we can assign

$$N_{\text{NR}} = \left( 1 - \frac{\nabla^2}{8m_{\mathcal{N}}^2} \right) N$$

and therefore, we finally get

$$N = \left( 1 + \frac{\nabla^2}{8m_{\mathcal{N}}^2} \right) N_{\text{NR}}. \quad (\text{A.15})$$

Collecting all together, we obtain the non-relativistic Dirac spinor with relativistic corrections,

$$\mathcal{N}(x) = \left( \begin{array}{c} 1 + \frac{1}{8m_{\mathcal{N}}^2} \nabla^2 \\ \frac{1}{2m_{\mathcal{N}}} \boldsymbol{\sigma} \cdot \nabla \end{array} \right) N(x) + \mathcal{O}(Q^3). \quad (\text{A.16})$$

## B. Fierz identities

The Fierz identity for the Pauli matrices has the form

$$\sigma_{\alpha\beta}^i \sigma_{\gamma\delta}^i = 2\delta_{\alpha\delta} \delta_{\gamma\beta} - \delta_{\alpha\beta} \delta_{\gamma\delta} . \quad (\text{B.1})$$

From this, we can get the following relations

$$\sigma_{\alpha\beta}^i \sigma_{\gamma\delta}^i = \frac{1}{2} (3\delta_{\alpha\delta} \delta_{\gamma\beta} - \sigma_{\alpha\delta}^i \sigma_{\gamma\beta}^i) , \quad (\text{B.2})$$

$$\delta_{\alpha\beta} \sigma_{\gamma\delta}^i = \frac{1}{2} (\delta_{\alpha\delta} \sigma_{\gamma\beta}^i + \sigma_{\alpha\delta}^i \delta_{\gamma\beta} + i\epsilon_{ilm} \sigma_{\alpha\delta}^l \sigma_{\gamma\beta}^m) , \quad (\text{B.3})$$

$$\sigma_{\alpha\beta}^i \delta_{\gamma\delta} = \frac{1}{2} (\delta_{\alpha\delta} \sigma_{\gamma\beta}^i + \sigma_{\alpha\delta}^i \delta_{\gamma\beta} - i\epsilon_{ilm} \sigma_{\alpha\delta}^l \sigma_{\gamma\beta}^m) , \quad (\text{B.4})$$

$$\begin{aligned} \sigma_{\alpha\beta}^i \sigma_{\gamma\delta}^j = \frac{1}{2} & \left( \sigma_{\alpha\delta}^i \sigma_{\gamma\beta}^j + \sigma_{\alpha\delta}^j \sigma_{\gamma\beta}^i + \delta^{ij} \delta_{\alpha\delta} \delta_{\gamma\beta} - \delta^{ij} \sigma_{\alpha\delta}^l \sigma_{\gamma\beta}^l \right. \\ & \left. + i\epsilon_{ijl} \sigma_{\alpha\delta}^l \delta_{\gamma\beta} - i\epsilon_{ijl} \delta_{\alpha\delta} \sigma_{\gamma\beta}^l \right) . \end{aligned} \quad (\text{B.5})$$

## C. Axial coupling and an $\mathcal{O}(1)$ correction

By using the equivalence of the constituent quark model (CQM) and the Skyrmon in the large  $N_c$  limit [19], it has been argued [9, 10] that there is a universal sub-leading corrections to various axial couplings and magnetic couplings of nucleons. This enters some operators that originate from  $\bar{\mathcal{B}}\gamma \cdot F\mathcal{B}$  in (2.11). The correction involves an additive shift of type  $N_c \rightarrow N_c + 2$ , and proved to be important if one wishes to obtain phenomenologically viable pion axial couplings and anomalous magnetic moments, for example. Although it is not clear whether the D4-D8 holographic model, with its quenched nature, captures this automatically, it does makes to think about such a shift when we consider comparing numbers against data,

We anticipate the same shift of coefficient would affect other operators descended from the same  $\bar{\mathcal{B}}\gamma \cdot F\mathcal{B}$ , which will affect all leading couplings to axial vector mesons and all tensor couplings to vector mesons. Practically this can be achieved by multiplying the wavefunction overlap coefficients,  $B$ 's and  $C$ 's, by a factor  $(N_c + 2)/N_c = 5/3$  as

$$\begin{aligned} g_V^{(k)triplet} &= A_{2k-1} + 2 \left( \frac{5}{3} \right) C_{2k-1} , \\ g_A^{(k)triplet} &= 2 \left( \frac{5}{3} \right) C_{2k} + A_{2k} , \\ g_{dV}^{(k)triplet} &= 2 \left( \frac{5}{3} \right) B_{2k-1} , \\ g_A^{triplet} &= 4 \left( \frac{5}{3} \right) C_0 + 2A_0 \end{aligned} \quad (\text{C.1})$$

and other couplings  $g_V^{(k)singlet}$ ,  $g_A^{(k)singlet}$  and  $g_A^{singlet}$  remain unchanged. Here we record the resulting changes in the mesons-nucleon cubic couplings in Table 6 and the resulting values of the LECs in Table 7.

$k$	$A_{2k-1}$ $=g_V^{(k)singlet}$	$B_{2k-1}$	$C_{2k-1}$	$A_{2k}$ $=g_A^{(k)singlet}$	$C_{2k}$	$A_{2k-1}+2(\frac{5}{3})C_{2k-1}$ $=g_V^{(k)triplet}$	$2(\frac{5}{3})B_{2k-1}$ $=g_{dV}^{(k)triplet}$	$2(\frac{5}{3})C_{2k}+A_{2k}$ $=g_A^{(k)triplet}$
1	5.93	3.05	-0.409	1.12	2.35	4.57	10.2	8.94
2	-3.21	-1.75	1.01	-1.03	-1.98	0.155	-5.82	-7.62
3	1.25	0.756	-0.967	0.521	0.863	-1.98	2.52	3.40
4	-0.305	-0.222	0.481	-0.149	-0.188	1.30	-0.739	-0.777
5	0.0401	0.0392	-0.116	0.0206	0.0160	-0.345	0.131	0.0739

**Table 6:**  $N_c$  shifted couplings.  $A_n, C_n$  are dimensionless and  $B_n$  are of dimension  $M_{KK}^{-1}$ .

	$(I = 0)$	<i>From</i> $(I = 1)$	<i>Total</i>
$C_S$ ( $10^{-4}$ MeV $^{-2}$ )	1.44	-0.464	0.976
$C_T$ ( $10^{-4}$ MeV $^{-2}$ )	-0.0272	0.369	0.342
$C_1$ ( $10^{-9}$ MeV $^{-4}$ )	-0.270	0.0493	-0.221
$C_2$ ( $10^{-9}$ MeV $^{-4}$ )	0.159	-1.11	-0.951
$C_3$ ( $10^{-9}$ MeV $^{-4}$ )	-0.0414	0.0947	0.0533
$C_4$ ( $10^{-9}$ MeV $^{-4}$ )	0.00302	0.0505	-0.0475
$C_5$ ( $10^{-9}$ MeV $^{-4}$ )	-0.240	-0.185	-0.425
$C_6$ ( $10^{-9}$ MeV $^{-4}$ )	0.0311	-0.0923	-0.0612
$C_7$ ( $10^{-9}$ MeV $^{-4}$ )	-0.00603	0.616	0.610

**Table 7:** The low energy constants of  $N_c$  shifted results.

## References

- [1] S. Weinberg, “Nuclear forces from chiral Lagrangians,” Phys. Lett. **B251** (1990) 288-292.
- [2] R. Machleidt and D. R. Entem, “Chiral effective field theory and nuclear forces,” Phys. Rept. **503** (2011) 1 [arXiv:1105.2919 [nucl-th]].
- [3] G. Ecker, J. Gasser, A. Pich and E. de Rafael, “The Role of Resonances in Chiral Perturbation Theory,” Nucl. Phys. B **321** (1989) 311.
- [4] E. Epelbaum, U. G. Meissner, W. Gloeckle, C. Elster, “Resonance saturation for four nucleon operators,” Phys. Rev. **C65** (2002) 044001. [arXiv:nucl-th/0106007].
- [5] G. Gelmini, B. Ritzi, “Chiral effective Lagrangian description of bulk nuclear matter,” Phys. Lett. **B357** (1995) 431-434. [arXiv:hep-ph/9503480]; G. E. Brown, M. Rho, “From chiral mean field to Walecka mean field and kaon condensation,” Nucl. Phys. **A596** (1996) 503-514. [arXiv:nucl-th/9507028].
- [6] B. D. Serot, J. D. Walecka, “The Relativistic Nuclear Many Body Problem,” Adv. Nucl. Phys. **16** (1986) 1-327.
- [7] K. Saito, “The quark-meson coupling model and chiral symmetry,” AIP Conf. Proc. **1261** (2010) 238-243. [arXiv:1004.2763 [nucl-th]].
- [8] D. Montano, H. D. Politzer, M. B. Wise, “Charged pion condensation in the chiral limit,” Nucl. Phys. **B375** (1992) 507-526; C. -H. Lee, G. E. Brown, D. -P. Min, M. Rho, “An Effective chiral Lagrangian approach to kaon - nuclear interactions: Kaonic atom and kaon condensation,” Nucl. Phys. **A585** (1995) 401-449. [arXiv:hep-ph/9406311].

- [9] D. K. Hong, M. Rho, H. -U. Yee, P. Yi, “Chiral Dynamics of Baryons from String Theory,” *Phys. Rev.* **D76** (2007) 061901. [arXiv:hep-th/0701276].
- [10] D. K. Hong, M. Rho, H. -U. Yee, P. Yi, “Dynamics of baryons from string theory and vector dominance,” *JHEP* **0709** (2007) 063. [arXiv:0705.2632 [hep-th]].
- [11] Y. Kim, S. Lee, P. Yi, “Holographic Deuteron and Nucleon-Nucleon Potential,” *JHEP* **0904** (2009) 086. [arXiv:0902.4048 [hep-th]].
- [12] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2** (1998) 231-252. [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov, A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” *Phys. Lett.* **B428** (1998) 105-114. [arXiv:hep-th/9802109]; E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2** (1998) 253-291. [arXiv:hep-th/9802150].
- [13] G. W. Gibbons, K. -i. Maeda, “Black Holes and Membranes in Higher Dimensional Theories with Dilaton Fields,” *Nucl. Phys.* **B298** (1988) 741.
- [14] T. Sakai, S. Sugimoto, “Low energy hadron physics in holographic QCD,” *Prog. Theor. Phys.* **113** (2005) 843-882. [arXiv:hep-th/0412141]; T. Sakai, S. Sugimoto, “More on a holographic dual of QCD,” *Prog. Theor. Phys.* **114** (2005) 1083-1118. [arXiv:hep-th/0507073].
- [15] C. Ordonez, L. Ray, U. van Kolck, “The Two nucleon potential from chiral Lagrangians,” *Phys. Rev.* **C53** (1996) 2086-2105. [arXiv:hep-ph/9511380].
- [16] L. Girlanda, S. Pastore, R. Schiavilla, M. Viviani, “Relativity constraints on the two-nucleon contact interaction,” *Phys. Rev.* **C81** (2010) 034005. [arXiv:1001.3676 [nucl-th]].
- [17] E. Epelbaum, Doctoral thesis, “The nucleon nucleon interaction in a chiral effective field theory,” *Berichte des Forschungszentrum Jülich*, 3803 (2000).
- [18] M. E. Luke, A. V. Manohar, “Reparametrization invariance constraints on heavy particle effective field theories,” *Phys. Lett.* **B286** (1992) 348-354. [hep-ph/9205228].
- [19] R. F. Dashen, E. E. Jenkins, A. V. Manohar, “The  $1/N(c)$  expansion for baryons,” *Phys. Rev.* **D49** (1994) 4713. [hep-ph/9310379]; “Spin flavor structure of large  $N(c)$  baryons,” *Phys. Rev.* **D51** (1995) 3697-3727. [hep-ph/9411234]; A. Hosaka, N. R. Walet, “Algebraic method for large- $N(c)$  QCD,” *Austral. J. Phys.* **50** (1997) 211-220.
- [20] V. B. Berestetskii, E. M. Lifshitz, L. P. Pitaevskii, “Quantum Electrodynamics,” Oxford, UK: Pergamon (1982) 652 P. (Course Of Theoretical Physics, 4).